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FREE FIELD CONSTRUCTION OF D-BRANES IN $N = 2$ SUPERCONFORMAL MINIMAL MODELS.

S. E. Parkhomenko

Landau Institute for Theoretical Physics
142432 Chernogolovka, Russia

spark@itp.ac.ru

Abstract

The construction of D-branes in $N = 2$ superconformal minimal models based on free-field realization of $N = 2$ super-Virasoro algebra unitary modules is represented.

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0. Introduction

The role of D -branes [1] in the description of certain nonperturbative degrees of freedom of strings is by now well established and the study of their dynamics has lead to many new insights into string and M -theory [2], [3]. Much of this study was done in the large volume regime where geometric techniques provide reliable information. The extrapolation into the stringy regime usually requires boundary conformal field theory (CFT) methods. In this approach D -brane configurations are given by conformally invariant boundary states or boundary conditions. However a complete microscopic description these configurations are well understood only for the case of flat and toric backgrounds where the CFT on the world sheet is a theory of free fields. Due to this reason boundary state formalism for D -branes has been subsequently developed and many calculations concerning the scattering and couplings of closed strings in a D -brane background have been given exactly in [4]- [7], [8].

The class of models of rational CFT gives the examples of curved string backgrounds where the construction of the boundary states leaving a whole chiral symmetry algebra unbroken can be given in principle and the interaction of these states with closed strings can be calculated exactly. But in practice the calculation of closed string amplitudes in general CFT backgrounds is available only if the corresponding free field realization of the model is known. Therefore, it is important to extend free field approach to the case of rational models of CFT with a boundary.

This problem has been treated recently to the case of $SU(2)$ WZNW model in [9], where the

Wakimoto free field realization of $sl(2)$ Kac-Moody algebra [10]- [14] has been used to boundary states construction.

$N = 2$ superconformal minimal models represent a subclass of rational CFT which is most important in the string theory applications. They are building blocks in Gepner models [15] which give an exact solution by CFT methods of the problem of string propagation on Calabi-Yau manifolds. The $N = 2$ minimal models as suggested are fixed points of $N = 2$ supersymmetric Landau-Ginzburg (LG) models [16], [17]. This rigorously justified suggestion gives a possibility to describe string propagation on Calabi-Yau manifold also in geometric terms and provides a link between the algebraic structure encoded in $N = 2$ minimal models and geometry of the manifold. These mutually complementary approaches of CFT and LG are very efficient also in investigation of D -branes on Calabi-Yau manifolds [18]- [25].

The free field realization of the unitary $N = 2$ super-Virasoro algebra representations has been developed by Feigin and Semikhatov in [28]. From the one hand, it gives an efficient way for correlation functions calculation. From the other hand it is closely related [27], [28] with Wakimoto free field description of $SU(2)$ WZNW model as well as with LG approach to $N = 2$ minimal models. The effect of boundaries has already been studied in LG approach [23], [24] and in the context of coset model [26]. Thus, it is important to extend the free field realization to the case of $N = 2$ minimal models with boundaries.

In this note we extend the construction of [28] to the case of $N = 2$ minimal models with boundaries. In section 1, we review the free field realization and butterfly resolution [28] of Feigin and Semikhatov of the unitary modules in $N = 2$ minimal models and obtain free field representation for the characters. In section 2, we construct Ishibashi states of A, B -types in Fock modules. In section 3, we obtain in an explicit form A, B -type Ishibashi state for each irreducible $N = 2$ minimal model module using superposition of Ishibashi states of Fock modules from butterfly resolution. The coefficients of the superposition are fixed by imposing BRST invariance condition which is similar to that of the bulk theory. At the end of the section free field construction of boundary states is represented using Cardy's prescription. In the last section we briefly discuss free field representation for boundary correlations functions as well as some generalizations of the construction.

1. Free-field realization of $N = 2$ minimal models irreducible representations.

In this section we briefly discuss free-field construction of Feigin and Semikhatov [28] of the irreducible modules in $N = 2$ superconformal minimal models.

1.1. Free-field representations of $N = 2$ super-Virasoro algebra.

We introduce (in the left-moving sector) the free bosonic fields $X(z), X^*(z)$ and free fermionic fields $\psi(z), \psi^*(z)$, so that its singular OPE's are given by

$$\begin{aligned} X^*(z_1)X(z_2) &= \ln(z_{12}) + reg., \\ \psi^*(z_1)\psi(z_2) &= z_{12}^{-1} + reg, \end{aligned} \tag{1}$$

where $z_{12} = z_1 - z_2$. Then for an arbitrary number μ the currents of $N = 2$ super-Virasoro algebra are given by

$$G^+(z) = \psi^*(z)\partial X(z) - \frac{1}{\mu}\partial\psi^*(z), \quad G^-(z) = \psi(z)\partial X^*(z) - \partial\psi(z),$$

$$\begin{aligned}
J(z) &= \psi^*(z)\psi(z) + \frac{1}{\mu}\partial X^*(z) - \partial X(z), \\
T(z) &= \partial X(z)\partial X^*(z) + \frac{1}{2}(\partial\psi^*(z)\psi(z) - \psi^*(z)\partial\psi(z)) - \\
&\quad \frac{1}{2}(\partial^2 X(z) + \frac{1}{\mu}\partial^2 X^*(z)),
\end{aligned} \tag{2}$$

and the central charge is

$$c = 3(1 - 2\frac{1}{\mu}). \tag{3}$$

As usual, the fermions in NS sector are expanded into half-integer modes:

$$\begin{aligned}
\psi(z) &= \sum_{r \in 1/2 + \mathbb{Z}} \psi[r]z^{-\frac{1}{2}-r}, \quad \psi^*(z) = \sum_{r \in 1/2 + \mathbb{Z}} \psi^*[r]z^{-\frac{1}{2}-r}, \\
G^\pm(z) &= \sum_{r \in 1/2 + \mathbb{Z}} G^\pm[r]z^{-\frac{3}{2}-r},
\end{aligned} \tag{4}$$

and they are expanded into integer modes in R sector:

$$\begin{aligned}
\psi(z) &= \sum_{r \in \mathbb{Z}} \psi[r]z^{-\frac{1}{2}-r}, \quad \psi^*(z) = \sum_{r \in \mathbb{Z}} \psi^*[r]z^{-\frac{1}{2}-r}, \\
G^\pm(z) &= \sum_{r \in \mathbb{Z}} G^\pm[r]z^{-\frac{3}{2}-r}.
\end{aligned} \tag{5}$$

The bosons $X(z), X^*(z), J(z), T(z)$ are expanded in both sectors into integer modes:

$$\begin{aligned}
\partial X(z) &= \sum_{n \in \mathbb{Z}} X[n]z^{-1-n}, \quad \partial X^*(z) = \sum_{n \in \mathbb{Z}} X^*[n]z^{-1-n}, \\
J(z) &= \sum_{n \in \mathbb{Z}} J[n]z^{-1-n}, \quad T(z) = \sum_{n \in \mathbb{Z}} L[n]z^{-2-n}.
\end{aligned} \tag{6}$$

To describe the modules of $N = 2$ Virasoro superalgebra in NS sector we define the vacuum state $|p, p^* >$ such that

$$\begin{aligned}
\psi[r]|p, p^* > &= \psi^*[r]|p, p^* > = 0, r \geq \frac{1}{2}, \\
X[n]|p, p^* > &= X^*[n]|p, p^* > = 0, n \geq 1, \\
X[0]|p, p^* > &= p|p, p^* >, \quad X^*[0]|p, p^* > = p^*|p, p^* >.
\end{aligned} \tag{7}$$

It is a primary state with respect to the $N = 2$ Virasoro algebra

$$\begin{aligned}
G^\pm[r]|p, p^* > &= 0, r > 0, \\
J[n]|p, p^* > &= L[n]|p, p^* > = 0, n > 0, \\
J[0]|p, p^* > &= \frac{j}{\mu}|p, p^* > = 0, \\
L[0]|p, p^* > &= \frac{h(h+2) - j^2}{4\mu}|p, p^* > = 0,
\end{aligned} \tag{8}$$

where $j = p^* - \mu p$, $h = p^* + \mu p$. The vacuum state $|p, p^* \rangle$ corresponds to the vertex operator $V_{(p, p^*)}(z) \equiv \exp(pX^*(z) + p^*X(z))$ placed at $z = 0$.

We denote by F_{p, p^*} the Fock module generated from the vector $|p, p^* \rangle$ by the fermionic operators $\psi^*[r]$, $\psi[r]$, $r < \frac{1}{2}$, and bosonic operators $X^*[n]$, $X[n]$, $n < 0$.

It is easy to calculate the character $\omega_{p, p^*}(q, u)$ of the Fock module F_{p, p^*} . By the definition

$$\omega_{p, p^*}(q, u) = \text{Tr}_{F_{p, p^*}}(q^{L[0] - \frac{c}{24}} u^{J[0]}). \quad (9)$$

Thus we obtain

$$\omega_{p, p^*}(q, u) = q^{\frac{h(h+2)-j^2}{4\mu} - \frac{c}{24}} u^{\frac{j}{\mu}} \frac{\Theta(q, u)}{\eta(q)^3}, \quad (10)$$

where we have used the Jacobi theta-function

$$\Theta(q, u) = q^{\frac{1}{8}} \sum_{m \in \mathbb{Z}} q^{\frac{1}{2}m^2} u^{-m} \quad (11)$$

and the Dedekind eta-function

$$\eta(q) = q^{\frac{1}{24}} \prod_{m=1}^{\infty} (1 - q^m). \quad (12)$$

The $N = 2$ Virasoro algebra has the following set of automorphisms which is known as spectral flow [30]

$$\begin{aligned} G^\pm[r] &\rightarrow G_t^\pm[r] \equiv G^\pm[r \pm t], \\ L[n] &\rightarrow L_t[n] \equiv L[n] + tJ[n] + t^2 \frac{c}{6} \delta_{n,0}, \quad J[n] \rightarrow J_t[n] \equiv J[n] + t \frac{c}{3} \delta_{n,0}, \end{aligned} \quad (13)$$

where $t \in \mathbb{Z}$. Note that spectral flow is intrinsic property of $N = 2$ super-Virasoro algebra and hence, it does not depend on a particular realization. Allowing in (13) t to be half-integer, we obtain the isomorphism between the NS and R sectors.

The spectral flow action on the free fields can be easily described if we bosonise fermions ψ^*, ψ

$$\psi(z) = \exp(-y(z)), \quad \psi^*(z) = \exp(+y(z)). \quad (14)$$

and introduce spectral flow vertex operator [29]

$$U_t(z) = \exp(-t(y + \frac{1}{\mu} X^* - X)(z)). \quad (15)$$

The following OPE's

$$\begin{aligned} \psi(z_1)U_t(z_2) &= z_{12}^t : \psi(z_1)U_t(z_2) :, \\ \psi^*(z_1)U_t(z_2) &= z_{12}^{-t} : \psi^*(z_1)U_t(z_2) :, \\ \partial X^*(z_1)U_t(z_2) &= z_{12}^{-1} t U_t(z_2) + r., \\ \partial X(z_1)U_t(z_2) &= -z_{12}^{-1} \frac{t}{\mu} U_t(z_2) + r. \end{aligned} \quad (16)$$

give the action of spectral flow on the modes of the free fields

$$\begin{aligned} \psi[r] &\rightarrow \psi[r - t], \quad \psi^*[r] \rightarrow \psi^*[r + t], \\ X^*[n] &\rightarrow X^*[n] + t\delta_{n,0}, \quad X[n] \rightarrow X[n] - \frac{t}{\mu} \delta_{n,0}. \end{aligned} \quad (17)$$

The action of the spectral flow on the vertex operator $V_{(p,p^*)}(z)$ is given by the normal ordered product of the vertex $U_t(z)$ and $V_{p,p^*}(z)$.

1.2. Irreducible $N = 2$ super-Virasoro representations and butterfly resolution.

The $N = 2$ minimal models are characterized by the condition that μ is integer and $\mu \geq 2$. In NS sector the irreducible highest-weight modules, constituting the (left-moving) space of states of the minimal model, are unitary and labeled by two integers h, j , where $h = 0, \dots, \mu - 2$ and $j = -h, -h + 2, \dots, h$. The highest-weight vector $|w_{h,j} \rangle$ of the module satisfies the conditions (which are similar to (8))

$$\begin{aligned} G^\pm[r]|w_{h,j} \rangle &= 0, r > 0, \\ J[n]|w_{h,j} \rangle &= L[n]|w_{h,j} \rangle = 0, n > 0, \\ J[0]|w_{h,j} \rangle &= \frac{j}{\mu}|w_{h,j} \rangle, \\ L[0]|w_{h,j} \rangle &= \frac{h(h+2) - j^2}{4\mu}|w_{h,j} \rangle. \end{aligned} \quad (18)$$

If in addition to the conditions (18) the relation

$$G^+[-1/2]|w_{h,j} \rangle = 0 \quad (19)$$

is satisfied we call the vector $|w_{h,j} \rangle$ and the module $M_{h,j}$ chiral highest-weight vector (chiral primary state) and chiral module, correspondingly. In this case we have $h = j$. Analogously, anti-chiral highest-weight vector (anti-chiral primary state) and anti-chiral module can be defined if instead of (19)

$$G^-[-1/2]|w_{h,j} \rangle = 0 \quad (20)$$

is satisfied. In this case $h = -j$. In [28] the highest-weight vectors satisfying (18) are called massive highest-weight vectors and the vectors satisfying in addition to (19) are called topological highest-weight vectors. In this paper we prefer to use the terms introduced in [29].

As we have seen in the preceding subsection, the highest weight vectors (18) can be realized by the Fock vacuum vectors $|p, p^* \rangle$. But the corresponding Fock modules are reducible with respect to $N = 2$ super-Virasoro algebra. To construct irreducible representations one needs to introduce the integer lattice of the momentums:

$$P = \{(p, p^*) | p, p^* \in \mathbb{Z}\} \quad (21)$$

and the space

$$F_P = \oplus_{(p,p^*) \in P} F_{p,p^*}. \quad (22)$$

Following [28] we introduce two fermionic screening currents $S^\pm(z)$ and the charges Q^\pm of the currents

$$\begin{aligned} S^+(z) &= \psi^* \exp(X^*)(z), \quad S^-(z) = \psi \exp(\mu X)(z), \\ Q^\pm &= \oint dz S^\pm(z) \end{aligned} \quad (23)$$

These charges commute with the generators of $N = 2$ super-Virasoro algebra (2) and act in the space F_P . Moreover they are nilpotent and mutually anticommute

$$(Q^+)^2 = (Q^-)^2 = \{Q^+, Q^-\} = 0. \quad (24)$$

Due to these properties one can combine the charges Q^\pm into BRST operator acting in F_P and build a BRST complex consisting of Fock modules $F_{p,p^*} \in F_P$ such that its cohomology is given by NS sector $N = 2$ minimal model irreducible module $M_{h,j}$. This complex has been constructed in [28].

Let us consider first free field construction for the chiral module $M_{h,h}$. In this case the complex (which is known due to Feigin and Semikhatov as butterfly resolution) can be represented by the following diagram

$$\begin{array}{ccccccc}
& \vdots & & \vdots & & & \\
& \uparrow & & \uparrow & & & \\
\cdots & \leftarrow & F_{1,h+\mu} & \leftarrow & F_{0,h+\mu} & & \\
& \uparrow & & \uparrow & & & \\
\cdots & \leftarrow & F_{1,h} & \leftarrow & F_{0,h} & & \\
& & & \swarrow & & & \\
& & & & F_{-1,h-\mu} & \leftarrow & F_{-2,h-\mu} & \leftarrow & \cdots \\
& & & & \uparrow & & \uparrow & & \\
& & & & F_{-1,h-2\mu} & \leftarrow & F_{-2,h-2\mu} & \leftarrow & \cdots \\
& & & & \uparrow & & \uparrow & & \\
& & & & \vdots & & \vdots & &
\end{array}
\tag{25}$$

The horizontal arrows in this diagram are given by the action of Q^+ and vertical arrows are given by the action of Q^- . The diagonal arrow at the middle of butterfly resolution is given by the action of Q^+Q^- (which equals $-Q^-Q^+$ due to (24)). Ghost number operator g of this complex is defined for an arbitrary vector $|v_{n,m}\rangle \in F_{n,m\mu+h}$ by

$$\begin{aligned}
g|v_{n,m}\rangle &= (n+m)|v_{n,m}\rangle, \text{ if } n, m \geq 0, \\
g|v_{n,m}\rangle &= (n+m+1)|v_{n,m}\rangle, \text{ if } n, m < 0.
\end{aligned}
\tag{26}$$

For an arbitrary vector of the complex $|v_N\rangle$ with the ghost number N the differential d_N is defined by

$$\begin{aligned}
d_N|v_N\rangle &= (Q^+ + Q^-)|v_N\rangle, \text{ if } N \neq -1, \\
d_N|v_N\rangle &= Q^+Q^-|v_N\rangle, \text{ if } N = -1.
\end{aligned}
\tag{27}$$

and rises the ghost number by 1.

Theorem 1.1.

[28]. Complex (25) is exact except at the $F_{0,h}$ module, where the cohomology is given by the chiral module $M_{h,h}$.

The butterfly resolution allows to write the character $\chi_h(q, u) \equiv \text{Tr}_{M_{h,h}}(q^{L[0] - \frac{c}{24}} u^{J[0]})$ of the module $M_{h,h}$ as an alternated sum:

$$\begin{aligned}
\chi_h(q, u) &= \chi_h^{(l)}(q, u) - \chi_h^{(r)}(q, u), \\
\chi_h^{(l)}(q, u) &= \sum_{n,m \geq 0} (-1)^{n+m} \omega_{n,h+m\mu}(q, u), \\
\chi_h^{(r)}(q, u) &= \sum_{n,m > 0} (-1)^{n+m} \omega_{-n,h-m\mu}(q, u),
\end{aligned}
\tag{28}$$

where $\chi_h^{(l)}(q, u)$ and $\chi_h^{(r)}(q, u)$ are the characters of the left and right wings of the resolution.

To obtain the resolutions for other (anti-chiral and non-chiral) modules we note first that all irreducible modules can be obtained from the chiral modules $M_{h,h}$, $h = 0, \dots, \mu - 2$ by the spectral flow action [31]. It turns out that one can get all the resolutions by the spectral flow action also. Indeed, the charges Q^\pm commute with spectral flow operator U_t as it is easy to see from the corresponding OPE's, hence, the resolutions in NS sector are generated from (25) by the operators U_t , $t = -h, -h + 1, \dots, -1$. The resolutions in R sector are generated from the resolutions in NS sector by the spectral flow operator $U_{-\frac{1}{2}}$.

To illustrate how it works we consider the Fock module $F_{0,h}$ from the middle of the resolution. The vector $|0, h\rangle$, ($h > 0$) represents $N = 2$ super-Virasoro algebra chiral highest-weight vector $|w_{h,h}\rangle$ of the module $M_{h,h}$. The vector $\psi[-1/2]|0, h\rangle = \frac{1}{h}G^-[-1/2]|0, h\rangle$ represents a cohomology class, hence it is in irreducible module $M_{h,h}$. Acting on this vector by the operator U_{-1} we obtain the vector $|\frac{1}{\mu}, h-1\rangle$, which is $N = 2$ super-Virasoro algebra highest-weight vector $|w_{h,h-2}\rangle$ of the module $M_{h,h-2}$. Analogously, the vector $\psi[-3/2]\psi[-1/2]|0, h\rangle = \frac{1}{h(h-1)}G^-[-3/2]G^-[-1/2]|0, h\rangle$ represents another vector from irreducible module $M_{h,h}$. Acting on this vector by U_{-2} we obtain the vector $|\frac{2}{\mu}, h-2\rangle$, which is $N = 2$ super-Virasoro algebra highest-weight vector $|w_{h,h-4}\rangle$ of the module $M_{h,h-4}$. Going by this way further, we arrive at the end the vector $|\frac{h}{\mu}, 0\rangle = U_{-h}\frac{1}{h!}G^-[-1/2-h]\dots G^-[-1/2]|0, h\rangle$, which is anti-chiral highest-weight vector $|w_{h,-h}\rangle$ of the anti-chiral module $M_{h,-h}$. The modules from R sector can be generated analogously.

2. Ishibashi states in the Fock modules of $N = 2$ super- Virasoro algebra.

In this section we begin to develop free field representation of $N = 2$ minimal models Ishibashi states. Thus, it will be implied in what follows that the right-moving sector of the model is realized by the free-fields $\bar{X}(\bar{z})$, $\bar{X}^*(\bar{z})$, $\bar{\psi}(\bar{z})$, $\bar{\psi}^*(\bar{z})$, and the right-moving $N = 2$ super-Virasoro algebra is given by the formulas similar to (2).

There are two types of boundary states preserving $N = 2$ super-Virasoro algebra [32], usually called B -type

$$\begin{aligned} (L[n] - \bar{L}[-n])|B\rangle\rangle &= (J[n] + \bar{J}[-n])|B\rangle\rangle = 0, \\ (G^+[r] + \eta\bar{G}^+[-r])|B\rangle\rangle &= (G^-[r] + \eta\bar{G}^-[-r])|B\rangle\rangle = 0 \end{aligned} \quad (29)$$

and A -type states

$$\begin{aligned} (L[n] - \bar{L}[-n])|A\rangle\rangle &= (J[n] - \bar{J}[-n])|A\rangle\rangle = 0, \\ (G^+[r] + \eta\bar{G}^-[-r])|A\rangle\rangle &= (G^-[r] + \eta\bar{G}^+[-r])|A\rangle\rangle = 0 \end{aligned} \quad (30)$$

where $\eta = \pm 1$. The Ishibashi states (as well as the boundary states) can be considered as the linear functionals on the space of states of the $N = 2$ minimal model. From the other hand, we have seen in the Sec.1, that the space of states in the left- moving sector is represented by the cohomology groups of butterfly resolutions. It is clear that similar construction can be applied for the right-moving space of states. Therefore, free field construction of Ishibashi states has to be consistent with the resolutions. This problem of consistence will be postponed to the next section. In this section we consider the most simple solutions of (29), (30) in the tensor product of the left-moving Fock module F_{p,p^*} and right-moving Fock module $\bar{F}_{\bar{p},\bar{p}^*}$. We shall call these states as linear Ishibashi [33] states and denote by $|p, p^*, \bar{p}, \bar{p}^*, \eta, B(A)\rangle\rangle$.

Let us consider B -type linear Ishibashi states in NS sector. They can be easily obtained from the following ansatz for fermions

$$\begin{aligned}(\psi^*[r] - \imath a \bar{\psi}^*[-r])|p, p^*, \bar{p}, \bar{p}^*, \eta, B \rangle \rangle &= 0, \\(\psi[r] - \imath b \bar{\psi}[-r])|p, p^*, \bar{p}, \bar{p}^*, \eta, B \rangle \rangle &= 0\end{aligned}\quad (31)$$

where a, b are the arbitrary constants. Substituting these relations into (29) and using (2) we find

$$\begin{aligned}a &= b = \eta, \\ \bar{p} &= -p - \frac{1}{\mu}, \quad \bar{p}^* = -p^* - 1, \\ (X[n] + \bar{X}[-n] + \frac{1}{\mu}\delta_{n,0})|p, p^*, \bar{p}, \bar{p}^*, \eta, B \rangle \rangle &= 0, \\ (X^*[n] + \bar{X}^*[-n] + \delta_{n,0})|p, p^*, \bar{p}, \bar{p}^*, \eta, B \rangle \rangle &= 0.\end{aligned}\quad (32)$$

Thus, the linear B -type Ishibashi state in NS sector is given by the standard expression [5], [34], [35]

$$\begin{aligned}|p, p^*, \eta, B \rangle \rangle &= \prod_{n=1} \exp\left(-\frac{1}{n}(X^*[-n]\bar{X}[-n] + X[-n]\bar{X}^*[-n])\right) \\ \prod_{r=1/2} \exp(\imath \eta(\psi^*[-r]\bar{\psi}[-r] + \psi[-r]\bar{\psi}^*[-r]))|p, p^*, -p - \frac{1}{\mu}, -p^* - 1 \rangle &.\end{aligned}\quad (33)$$

The closed string cylinder amplitude between such states in NS sector is given by

$$\begin{aligned}\langle\langle p_2, p_2^*, \eta, B | q^{L[0]-c/24} u^{J[0]} | p_1, p_1^*, \eta, B \rangle \rangle &= \\ \delta(p_1 - p_2) \delta(p_1^* - p_2^*) \omega_{p_1, p_1^*}(q, u).\end{aligned}\quad (34)$$

Note that the state $\langle\langle p, p^*, \eta, B |$ is defined in such a way to satisfy conjugate boundary conditions and to take into account the charge asymmetry [36]- [38] of the free-field realization of the minimal model.

Introducing the new set of bosonic oscillators

$$\begin{aligned}v[n] &= \frac{\imath}{\sqrt{2\mu}}(X^*[n] - \mu X[n]), \\ u[n] &= \frac{1}{\sqrt{2\mu}}(X^*[n] + \mu X[n]),\end{aligned}\quad (35)$$

one can rewrite the B -type conditions (32) as

$$\begin{aligned}(v[n] + \bar{v}[-n])|p, p^*, \bar{p}, \bar{p}^*, \eta, B \rangle \rangle &= 0, \\ (u[n] + \bar{u}[-n] + \sqrt{\frac{2}{\mu}}\delta_{n,0})|p, p^*, \bar{p}, \bar{p}^*, \eta, B \rangle \rangle &= 0.\end{aligned}\quad (36)$$

One can consider the coordinates $(\exp(u), v)$ as the polar coordinates on a complex plane and think of the conditions (36) as Neumann along both of coordinates (u, v) . From this point

of view the bosonic part $\imath\sqrt{\frac{2}{\mu}}(v[0] + \bar{v}[0])$ of the sum $J[0] + \bar{J}[0]$ generates an action of a circle on the complex plane such that v is an angular coordinate. Then, B -type states correspond to 2-dimensional D -branes filling the complex plane. One can equally well to think of the conditions (36) as Dirichlet ones along (u, v) such that the circle action on the complex plane is generated by $\imath\sqrt{\frac{2}{\mu}}(v[0] - \bar{v}[0])$ and then, B -type boundary states correspond to $D0$ -branes. There are also two additional interpretations of the conditions when we consider one of the relations (36) as Neumann and another one as Dirichlet condition. Thus, one has four possibilities. As we shall see at the end of this section, mirror symmetry left only two of them

The linear A -type Ishibashi states can be found analogously. We start from the ansatz for fermions

$$\begin{aligned} (\psi^*[r] - \imath a \bar{\psi}[-r])|p, p^*, \bar{p}, \bar{p}^*, \eta, A \gg &= 0, \\ (\psi[r] - \imath b \bar{\psi}^*[-r])|p, p^*, \bar{p}, \bar{p}^*, \eta, A \gg &= 0 \end{aligned} \quad (37)$$

where a, b are the arbitrary constants. Then we find

$$\begin{aligned} a &= \eta\mu, \quad b = \frac{\eta}{\mu}, \\ \bar{p} &= -\frac{1+p^*}{\mu}, \quad \bar{p}^* = -\mu p - 1, \\ (\mu X[n] + \bar{X}^*[-n] + \delta_{n,0})|p, p^*, \bar{p}, \bar{p}^*, \eta, A \gg &= 0, \\ (X^*[n] + \mu \bar{X}[-n] + \delta_{n,0})|p, p^*, \bar{p}, \bar{p}^*, \eta, A \gg &= 0. \end{aligned} \quad (38)$$

Hence the linear A -type Ishibashi state (in NS sector) is given by

$$\begin{aligned} |p, p^*, \eta, A \gg &= \prod_{n=1} \exp\left(-\frac{1}{n}(\mu X[-n]\bar{X}[-n] + \frac{1}{\mu}X^*[-n]\bar{X}^*[-n])\right) \\ \prod_{r=1/2} \exp(\imath\eta(\frac{1}{\mu}\psi^*[-r]\bar{\psi}^*[-r] + \mu\psi[-r]\bar{\psi}[-r]))|p, p^*, -\frac{1+p^*}{\mu}, -\mu p - 1 &> \end{aligned} \quad (39)$$

and the corresponding closed string cylinder amplitude coincide with the (34).

One can find that (38) can be rewritten in (u, v) coordinates as

$$\begin{aligned} (v[n] - \bar{v}[-n])|p, p^*, \bar{p}, \bar{p}^*, \eta, A \gg &= 0, \\ (u[n] + \bar{u}[-n] + \sqrt{\frac{2}{\mu}}\delta_{n,0})|p, p^*, \bar{p}, \bar{p}^*, \eta, A \gg &= 0. \end{aligned} \quad (40)$$

One can consider these relations as Neumann condition along the coordinate u and Dirichlet condition along v . Then, A -type states correspond to 1-dimensional D -branes along the rays in complex plane which is in agreement with the results [23], [24] obtained in LG approach. But we are free to choose, similar to the case of B -type states, three other possible interpretations. Because of A -type Ishibashi state (39) can be obtained from B -type Ishibashi state (33) by the Mirror involution σ in the right-moving sector:

$$\begin{aligned} \sigma \bar{v}[n] &= -\bar{v}[n], \quad \sigma \bar{u}[n] = \bar{u}[n], \\ \sigma \bar{\psi}[r] &= \frac{1}{\mu}\bar{\psi}^*[r], \quad \sigma \bar{\psi}^*[r] = \mu\bar{\psi}[r] \end{aligned} \quad (41)$$

we are left with two possibilities for A -type states: $D1$ -branes along the rays or along the circles of the complex plane, and we have two possibilities for B -type states: $D2$ -branes or $D0$ -branes. But, Poincare duality relates to each other the states for each type, so that we are left with $D1$ -brane for A -type and $D0$ -brane for B -type.

3. Boundary states in $N = 2$ minimal models.

3.1. Ishibashi states in the irreducible modules of $N = 2$ super-Virasoro algebra.

In this section we represent free field construction of Ishibashi states of irreducible modules $M_{h,j}$. The construction uses the linear Ishibashi states (33), (39) as the building blocks in such a way to be consistent with butterfly resolutions of irreducible modules.

The relations (28), (34) indicate that Ishibashi state of irreducible module has to be given as a superposition of linear Ishibashi states of the Fock modules. Indeed, let us consider the following superposition of B -type free-field Ishibashi states

$$|M_{h,h}, \eta, B \rangle \rangle = \sum_{n,m \geq 0} c_{n,m} |n, m\mu + h, \eta, B \rangle \rangle + \sum_{n,m > 0} c_{-n,-m} |-n, -m\mu + h, \eta, B \rangle \rangle, \quad (42)$$

where the summation is performed over the momentums from the butterfly resolution (25). It is easy to see that this state satisfies the relations (29). The arbitrary coefficients $c_{n,m}$ and $c_{-n,-m}$ can be fixed partly from the condition that closed string cylinder amplitude between the Ishibashi states gives the characters of irreducible modules. In the free field realization it is equivalent to the relation

$$\langle \langle M_{h',h'}, \eta, B | (-1)^g q^{L[0]-c/24} u^{J[0]} | M_{h,h}, \eta, B \rangle \rangle = \delta_{h',h} \chi_h(q, u). \quad (43)$$

Indeed, using (34) one can find

$$\begin{aligned} & \langle \langle M_{h',h'}, \eta, B | (-1)^g q^{L[0]-c/24} u^{J[0]} | M_{h,h}, \eta, B \rangle \rangle = \\ & \delta_{h',h} \left(\sum_{n,m \geq 0} (-1)^{n+m} |c_{n,m}|^2 \omega_{n,h+m\mu}(q, u) - \sum_{n,m > 0} (-1)^{n+m} |c_{-n,-m}|^2 \omega_{-n,h-m\mu}(q, u) \right). \end{aligned} \quad (44)$$

Comparing with (28) we find

$$|c_{n,m}|^2 = |c_{-n,-m}|^2 = 1. \quad (45)$$

Thus, the state (42) is a good candidate for free-field realization (in NS sector) of B -type Ishibashi state of the chiral module $M_{h,h}$. It would be a genuine Ishibashi state if it did not radiate nonphysical closed string states which are present in the free field representation of the model. In other words, the overlap of this state with an arbitrary closed string state which does not belong to the Hilbert space of the $N = 2$ minimal model should vanish. As we will see this condition can be formulated as a $BRST$ invariance condition of the state (42) and it will fix the coefficients $c_{n,m}$, $c_{-n,-m}$ up to the common factor.

To formulate $BRST$ invariance condition one has to consider what kind of closed string states in the free field realization can interact with the state (42). They come from the product of left-moving and right-moving Fock modules $F_{n,h+m\mu} \otimes \bar{F}_{-n-\frac{1}{\mu}, -1-h-m\mu}$, where $n, m \geq 0$ or $n, m < 0$. The left-moving modules of the superposition (42) constitute the butterfly resolution (25) whose cohomology is given by the irreducible chiral module $M_{h,h}$. What about the Fock modules from the right-moving sector? They don't form the resolution like (25) due to the relations for the

momentums from (32) and (38). Instead the right-moving Fock modules constitute the dual butterfly resolution:

$$\begin{array}{ccccccc}
& & \vdots & & \vdots & & \\
& & \downarrow & & \downarrow & & \\
\cdots & \rightarrow & \bar{F}_{-1-\frac{1}{\mu}, -1-h-\mu} & \rightarrow & \bar{F}_{-\frac{1}{\mu}, -1-h-\mu} & & \\
& & \downarrow & & \downarrow & & \\
\cdots & \rightarrow & \bar{F}_{-1-\frac{1}{\mu}, -1-h} & \rightarrow & \bar{F}_{-\frac{1}{\mu}, -1-h} & & \\
& & & \searrow & & & \\
& & & & \bar{F}_{1-\frac{1}{\mu}, -1-h+\mu} & \rightarrow & \bar{F}_{2-\frac{1}{\mu}, -1-h+\mu} \rightarrow \cdots \\
& & & & \downarrow & & \downarrow \\
& & & & \bar{F}_{1-\frac{1}{\mu}, -1-h+2\mu} & \rightarrow & \bar{F}_{2-\frac{1}{\mu}, -1-h+2\mu} \rightarrow \cdots \\
& & & & \downarrow & & \downarrow \\
& & & & \vdots & & \vdots
\end{array} \tag{46}$$

The arrows on this diagram are given by the same operators as on the diagram (25). We define the ghost number operator \bar{g} of the complex similar to the ghost number operator of the complex (25). For an arbitrary vector $|\bar{v}_{n,m}\rangle \in \bar{F}_{n-\frac{1}{\mu}, m\mu-1-h}$ by

$$\begin{aligned}
\bar{g}|\bar{v}_{n,m}\rangle &= (n+m)|v_{n,m}\rangle, \text{ if } n, m \leq 0, \\
\bar{g}|v_{n,m}\rangle &= (n+m-1)|v_{n,m}\rangle, \text{ if } n, m > 0.
\end{aligned} \tag{47}$$

The differentials $\bar{d}_{\bar{N}}$ are defined similar to (27). This resolution can also be used for description of irreducible modules due to

Theorem 3.1.

Complex (46) is exact except at the $\bar{F}_{-\frac{1}{\mu}, -1-h}$ module, where the cohomology is given by the anti-chiral module $M_{h,-h}$.

The proof of this theorem is similar to that one is given in [28] for the complex (25).

Thus, the states which can interact with (42) come from the product of resolution (25) and (46). The tensor product of complexes (25) and (46) constitutes the complex

$$\cdots \rightarrow C_{h,h}^{-2} \rightarrow C_{h,h}^{-1} \rightarrow C_{h,h}^0 \rightarrow C_{h,h}^{+1} \rightarrow \cdots, \tag{48}$$

which is graded by the sum of the ghost numbers $g + \bar{g}$ and for an arbitrary ghost number i the space $C_{h,h}^i$ is given by the sum of products of the Fock modules from the resolution (25) and (46) such that $g + \bar{g} = i$. The differential D of the complex (48) is defined by the differentials d_N and $\bar{d}_{\bar{N}}$ of the complexes (25) and (46)

$$D_i |v_N \otimes \bar{v}_{\bar{N}}\rangle = |d_N v_N \otimes \bar{v}_{\bar{N}}\rangle + (-1)^N |v_N \otimes \bar{d}_{\bar{N}} \bar{v}_{\bar{N}}\rangle, \tag{49}$$

where $|v_N\rangle$ is an arbitrary vector from the complex (25) with ghost number N , while $|\bar{v}_{\bar{N}}\rangle$ is an arbitrary vector from the complex (46) with the ghost number \bar{N} and $N + \bar{N} = i$. It follows from the Theorems (1.1) and (3.1) that the cohomology \mathbf{H}^* of the complex (48) is nonzero only at grading 0 and it is given by the product of irreducible modules $M_{h,h} \otimes \bar{M}_{h,-h}$.

The Ishibashi state we are looking for can be considered as a linear functional on the Hilbert space of $N = 2$ superconformal minimal model, then it has to be an element of the homology group \mathbf{H}_* . Therefore, the $BRST$ invariance condition for the state can be formulated as follows.

Let us define the action of the differential D on the state $|M_{h,h}, \eta, B \rangle\rangle$ by the relation

$$\langle\langle D^* M_{h,h}, \eta, B | v_N \otimes \bar{v}_{\bar{N}} \rangle\rangle \equiv \langle\langle M_{h,h}, \eta, B | D_{N+\bar{N}} | v_N \otimes \bar{v}_{\bar{N}} \rangle\rangle, \quad (50)$$

where $v_N \otimes \bar{v}_{\bar{N}}$ is an arbitrary element from $C_{h,h}^{N+\bar{N}}$. Then, $BRST$ invariance condition means that

$$D^* |M_{h,h}, \eta, B \rangle\rangle = 0. \quad (51)$$

Theorem 3.2.

Superposition (42) satisfies $BRST$ invariance condition (51) if the coefficients $c_{n,m}$, $c_{-n,-m}$ of the superposition obey the equations

$$\begin{aligned} c_{n,m} &= c_{0,0}, \text{ if } n+m = 2k, \\ c_{n,m} &= -\eta c_{0,0}, \text{ if } n+m = 2k+1. \end{aligned} \quad (52)$$

Thus the coefficients depends on the ghost number g .

Proof. In view of (42, 33) and because of differential D rises the ghost number by 1, only $BRST$ images of the states $|v_N \otimes \bar{v}_{-1-N} \rangle \in C_{h,h}^{-1}$ have nonzero overlap with Ishibashi state $|M_{h,h}, \eta, B \rangle\rangle$. Thus, one needs to show that

$$\begin{aligned} &\langle\langle M_{h,h}, \eta, B | D_{-1} | v_N \otimes \bar{v}_{-1-N} \rangle\rangle = \\ &\langle\langle M_{h,h}, \eta, B | d_N | v_N \otimes \bar{v}_{-1-N} \rangle\rangle + (-1)^N \langle\langle M_{h,h}, \eta, B | \bar{d}_{-1-N} | v_N \otimes \bar{v}_{-1-N} \rangle\rangle = 0. \end{aligned} \quad (53)$$

It will be implied during the proof that the state $D_{-1} |v_N \otimes \bar{v}_{-1-N} \rangle$ corresponds to the field $(D_{-1}(v_N \otimes \bar{v}_{-1-N}))(z, \bar{z})$ which is placed at the center $z = \bar{z} = 0$ of the unit disk.

1) Let $N \geq 0$ and hence v_N and \bar{v}_{-1-N} belong to the left wings of resolutions (25) and (46). Using relations (31) and (32) as well as the definitions (27), (23) one can rewrite the first term from (53) as

$$\begin{aligned} &\langle\langle M_{h,h}, \eta, B | d_N | v_N \otimes \bar{v}_{-1-N} \rangle\rangle = \\ &\eta \langle\langle M_{h,h}, \eta, B | (\exp(X_0^* - \bar{X}_0^*) \bar{Q}^+ + \exp(\mu(X_0 - \bar{X}_0)) \bar{Q}^-) | v_N \otimes \bar{v}_{-1-N} \rangle\rangle, \end{aligned} \quad (54)$$

where X_0^* , \bar{X}_0^* , X_0 , \bar{X}_0 are the constant modes canonically conjugated to the momentums $X[0]$, $\bar{X}[0]$, $X^*[0]$, $\bar{X}^*[0]$. The operators $\exp(X_0^* - \bar{X}_0^*)$, $\exp(X_0 - \bar{X}_0)$ only shift the momentums in the components of the superposition $\langle\langle M_{h,h}, \eta, B |$. From the other hand we can decompose the vectors $|v_N \rangle$ and $|\bar{v}_{-1-N} \rangle$ according to their components in the Fock modules of the resolution

$$\begin{aligned} |v_N \rangle &= |y_{N,h} \rangle + |y_{N-1,h+\mu} \rangle + \dots + |y_{0,h+N\mu} \rangle, \\ |\bar{v}_{-1-N} \rangle &= |\bar{y}_{-1-N-\frac{1}{\mu}, -1-h} \rangle + |\bar{y}_{-N-\frac{1}{\mu}, -1-h+\mu} \rangle + \dots + |\bar{y}_{-\frac{1}{\mu}, -1-h+(N+1)\mu} \rangle \end{aligned} \quad (55)$$

Substitution of this decomposition into the each term of the equation (53) gives (52).

2) Let $N = -1$ and hence v_{-1} and \bar{v}_0 belong to the vertexes of the right wings of resolutions. Using relations (31) and (32) as well as the definitions (27), (23) one can rewrite the first term from (53) as

$$\begin{aligned} & \langle\langle M_{h,h}, \eta, B | d_{-1} | v_{-1} \otimes \bar{v}_0 \rangle\rangle = \\ & \langle\langle M_{h,h}, \eta, B | \exp(X_0^* - \bar{X}_0^*) \exp(\mu(X_0 - \bar{X}_0)) \bar{Q}^+ \bar{Q}^- | v_{-1} \otimes \bar{v}_0 \rangle\rangle. \end{aligned} \quad (56)$$

Substitution of this decomposition into the each term of the equation (53) gives the relation from (52) when $n = m = -1$.

3) Let $N < -1$ and hence v_N and \bar{v}_{-1-N} belong to the right wings of resolutions. This case can be treated similar to the case $N \geq 0$ and hence we obtain (52). It proves the theorem.

Note that $BRST$ -closed state (42, 52) is not $BRST$ -exact due to (43). Hence, it represents a homology class from \mathbf{H}_* and it is defined modulo $BRST$ -exact states satisfying (29). Note also that normalization phase $c_{0,0}$ can not be fixed by the $BRST$ invariance condition. We fix the normalization of each Ishibashi state by $c_{0,0} = 1$.

Due to the arguments from the Sec.1 we can obtain free field construction of the remainder $N = 2$ minimal model B -type Ishibashi states in NS sector applying to the Ishibashi state $|M_{h,h}, \eta, B \rangle\rangle$ the spectral flow operators $U_t \bar{U}_{-t}$, where $t = -h, -h+1, \dots, -1$. Then, the action by the spectral flow operator $U_{-\frac{1}{2}} \bar{U}_{\frac{1}{2}}$ on the Ishibashi states from the NS sector gives free field construction of B -type Ishibashi states in R sector. It is also clear that free field construction of A -type Ishibashi states in $N = 2$ minimal models can be obtained from B -type Ishibashi states by the Mirror involution (41).

3.2. Boundary states.

Free field representation of the boundary states in NS or R sector can be constructed by applying Cardy's prescription [39] to free field realized Ishibashi states.

Let us denote by $S_{(h,j),(h',j')}$ the S-matrix of modular transformation of the full characters of $N = 2$ minimal model in NS sector:

$$\begin{aligned} \chi_{h,j}(q, 0) &= \sum_{h',j'} S_{(h,j),(h',j')} \chi_{h',j'}(\tilde{q}, 0), \\ S_{(h,j),(h',j')} &= \frac{1}{\sqrt{2\mu}} \sin\left(\frac{\pi(h+1)(h'+1)}{\mu}\right) \exp(i\pi \frac{jj'}{\mu}). \end{aligned} \quad (57)$$

where $q = \exp(i2\pi\tau)$, $\tilde{q} = \exp(-i\frac{2\pi}{\tau})$. Then Cardy's formula in NS sector gives the following boundary states

$$|D_{h,j}, \eta, A \rangle\rangle = \sum_{h',j'} D_{(h,j),(h',j')} |M_{h',j'}, \eta, A \rangle\rangle, \quad (58)$$

while in R sector it gives

$$|D_{h,j}, \pm, A \rangle\rangle = \pm i \sum_{(h',j')} D_{(h,j),(h',j')} U_{\frac{1}{2}} \bar{U}_{\frac{1}{2}} |M_{h',j'}, \pm, A \rangle\rangle, \quad (59)$$

where

$$D_{(h,j),(h',j')} = \frac{S_{(h,j),(h',j')}}{\sqrt{S_{(0,0),(h',j')}}} \quad (60)$$

and $h' = 0, \dots, \mu - 2$, $j' = -h', -h' + 2, \dots, h'$.

Free field B -type boundary states can be obtained from A -type boundary states by the orbifold projection [26]. In NS sector we obtain

$$|D_{h,j}, \eta, B \rangle \rangle = \sum_{h'} D_{(h,j),(h',0)} |M_{h',0}, \eta, B \rangle \rangle, \quad (61)$$

while in R sector we have the following

$$|D_{h,j}, \pm, B \rangle \rangle = \pm \sum_{h'} D_{(h,j),(h',0)} U_{\frac{1}{2}} \bar{U}_{\frac{-1}{2}} |M_{h',0}, \pm, B \rangle \rangle, \quad (62)$$

where $h' = 0, \dots, \mu - 2$.

The Ishibashi states in the expressions (58), (59), (61), (62) correspond to the full characters of the $N = 2$ Virasoro superalgebra. Free field realization of the standard $A(B)$ -type Ishibashi states $|h, j, s, A(B) \rangle \rangle$, where $s = 0, 2$ in NS sector and $s = -1, 1$ in R sector is given by the relations

$$\begin{aligned} |M_{h,j}, +, A(B) \rangle \rangle &= |h, j, 0, A(B) \rangle \rangle + |h, j, 2, A(B) \rangle \rangle, \\ |M_{h,j}, -, A(B) \rangle \rangle &= |h, j, 0, A(B) \rangle \rangle - |h, j, 2, A(B) \rangle \rangle, \end{aligned} \quad (63)$$

in NS sector and it is given by

$$\begin{aligned} U_{\frac{1}{2}} \bar{U}_{\frac{1}{2}} |M_{h,j}, +, A(B) \rangle \rangle &= |h, j, -1, A(B) \rangle \rangle + |h, j, 1, A(B) \rangle \rangle, \\ U_{\frac{1}{2}} \bar{U}_{\frac{1}{2}} |M_{h,j}, -, A(B) \rangle \rangle &= |h, j, -1, A(B) \rangle \rangle - |h, j, 1, A(B) \rangle \rangle, \end{aligned} \quad (64)$$

in R sector.

Due to the brief discussion of boundary conditions in Sec.2 we have the following geometric interpretation of boundary states in $N = 2$ minimal model. A -type boundary states can be considered as $D1$ -branes along the rays or along the circles in the complex plane and they are Poincare dual to each other. This is in agreement with the results [23], [24] obtained in LG approach. B -type boundary states can be considered as $D0$ or $D2$ -branes which are also Poincare dual to each other.

4. Discussion

In this note we represented free field construction of Ishibashi and boundary states in $N = 2$ superconformal minimal models using free field realization of $N = 2$ super-Virasoro algebra unitary modules. Each Ishibashi state of the model is given by infinite superposition of linear Ishibashi states of Fock modules, forming butterfly resolution of irreducible representation of $N = 2$ super-Virasoro algebra. It is shown that coefficients of the superposition are fixed by the $BRST$ invariance condition and the Ishibashi state constructed this way is $BRST$ closed but not $BRST$ exact and represents thereby a homology class. We group these free field realized Ishibashi states into the boundary states of $N = 2$ minimal model using the solution found by Cardy. Due to $BRST$ invariance the boundary states do not radiate non-physical closed string states (which are present originally in the free field space of states). We found that B -type boundary states corresponds to $D0$ or $D2$ -branes in complex plane target space. A -type D-branes is natural to identify with rays or circles in complex plane. This identification is in agreement with the results [23], [24] obtained in LG approach but more detail investigation of D-brane geometry in the free field approach needs to be done. It would be interesting to find

geometric interpretation of the superposition of linear Ishibashi states involved into the free field construction of the D -branes. In the R sector it is a brane anti-brane system due to (52) and it would be interesting to find the interpretation of this superposition in the context of tachyon condensation [40].

We close with a brief discussion of some directions to develop. At first we would like to point out that our free field construction can be easily generalized to the case of orientifolds. The second problem is a free field construction for boundary correlation functions. This problem is rather technical in fact because all needed ingredients are known. Indeed, free field representation of the boundary states is obtained in the present note. The free field description of the irreducible representations, vertex operators and the screening charge generating quantum group structure of the conformal blocks (note that the fermionic screenings Q^\pm of the butterfly resolution have trivial braiding relations) are known from [28]. The screening charge is nothing else the standard Wakimoto screening charge (the one involved in the construction of Felder-type resolution [10]) of $SU(2)$ WZNW model. In terms of the free fields it is given by the integral of the (bosonic) screening current:

$$Q_W = \oint dz S_W(z), \quad S_W = (\partial X^* + \psi^* \psi) \exp\left(-\frac{1}{\mu} X^* - X\right). \quad (65)$$

It is easy to check that Q_W commutes with $N = 2$ super-Virasoro algebra and fermionic charges Q^\pm . We hope to develop free field representation of the boundary correlation functions in future publication.

The third interesting direction is the generalization of our boundary state construction to the case of nondiagonal modular invariant partition functions. It is obviously important to generalize the free field construction of boundary states to the case of Gepner models.

Free field approach is also applies to $N = 2$ superconformal models with W -algebra of symmetries. Some of them is believed to coincide with Kazama-Suzuki models [41]. $N = 2$ minimal models is the simplest example of this situation when Kazama-Suzuki model is $SU(2) \times U(1)/U(1)$ coset and it would be interesting to extend free field construction of D -branes to more general class of Kazama-Suzuki models.

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